CATEGORY THEORY FOR ECONOMIC ANALYSIS

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To embrace a complicated subject by a conceptual framework helps formulate a mathematical problem and eventually find the best technical tools to solve practical tasks.

One of such practical tasks is to create effective software-based management systems, which are essential for every business, from a single enterprise to larger bodies of economics. The early attempts to use category theory (CT) outside of pure mathematics were made by D. Spivak from MIT (1), whom we follow believing that the language and toolset of CT can be useful throughout science, but we aim to add to this a wide range of practical tasks in economics.

In our previous work (2) we have described the economic cycle as a basic chain of categories: “Human Values” - “Consumer Needs” - “Visions of goods and services” - “Technological and Design constructs” - “Production infrastructure” - “Goods and Services” - “Consumer Market”, completing the permanent cycle. To demonstrate that besides obvious terminology adequacy there is deep essential validity of categorial approach we need to apply every mathematical statement, definition, theorem and basic conclusions to economic objects and their properties. First, we made sure that every economic entity fully corresponds to category of *sets* (**Set**) with elements of various nature, which is essential to apply the concept to a chain of categories transformations (represented by *functors*). The major prerequisite for CT applicability to economic chain of transformations is that they are “natural” by definition, preserving all arrows (functions, interlinks and dependencies) between elements. For economic categories sequentially undergoing natural transformations, all CT features (theorems and conclusions) are valid. Which is more, some of the most abstract CT notions and concepts find quite useful interpretations. To name a few: the initial and final objects and arrows make pairs within adjoining economic categories, binding together the links of chain. The other example of critical importance is how well Yoneda’s lemma (central for CT) could be used to analyze economic categories. For any object c within economic category C and functor I such as $I{:}C\rightarrow\mathbf{Set}$ there is a natural bijection (“*one-to-one distinct mapping*”): $Hom_{C\text{-}\mathbf{Set}}(Y_C,I) \rightarrow\approx I(c)$

Lemma can be reformulated based on important notion of *representative functor* which generates a new category of links between elements, conveying every aspect of c attributes, essential in C. As an example, any goods or service production pulls the whole sum of technologies to support it. It means the *brick* and *electronic chip* are produced in identical technological networks (as a map), but with different weights of links.

References.

1. *David I. Spivak* Category Theory for the Sciences. - MIT Press, Cambridge 2013.-268pp.
2. *Н.В. Зеликин* Теория категорий как понятийный аппарат для исследования ряда задач экономики. Тезисы докладов XXIV конференции МКО. - Москва 2017